Optical Cloak of Invisibility

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Abstract: We present the design and analysis of an optical cloak of invisibility with non-magnetic metamaterials. The general recipe for the implementation of such a device is provided. The cloaking performance is illustrated with finite-element simulations.

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1. Introduction

The electromagnetic cloak of invisibility based on coordinate transformation has opened a new door for the applications of metamaterials [1,2]. Unlike other cloaking methods, which are typically limited to sub-wavelength objects, the transformation approach allows the design of cloaking devices to render a macroscopic object invisible, and the design is not sensitive to the object that is being cloaked. The first experimental demonstration of such a cloak of cylindrical geometry at microwave frequencies was recently reported [3], where the cloaking was achieved by varying the dimensions of a series of split ring resonators (SRRs) to yield a desired gradient of the permeability in the radial direction. We note, however, that due to the limits of size scaling and the resonant losses in SRR-like structures, the design used in microwave cloaking [3] cannot be implemented for an optical cloak, which is certainly of particular interest because optical frequencies are where the word "invisibility" is conventionally defined. Here we present the design of a non-magnetic cloak operating at optical frequencies. The principle and structure of the proposed cylindrical cloak are analyzed, and the general recipe for the implementation of such a device is provided.

2. Design and analysis

The coordinate transformation used in the proposed non-magnetic optical cloak of cylindrical geometry is similar to that in [3], by which a cylindrical region $r<b$ is compressed into a concentric cylindrical shell $a<r<b$ as shown in Fig. 1a. In sharp contrast to the reported design of a microwave cloak with TE polarization [3], we focus on TM incidence with the magnetic field polarized along the z axis. In this case only μ_z, ε_r and ε_θ are relevant, and the transformation requires the following anisotropic permittivity and permeability in the cloaking shell:

$$\varepsilon_r = \frac{r-a}{r},\quad \varepsilon_\theta = \frac{r}{r-a},\quad \mu_z = \left(\frac{b}{b-a}\right)^2 \frac{r-a}{r}. \qquad (1)$$

It is worth noting that under TM illumination only one component of μ is of interest, which allows us to completely remove the need for any optical magnetism. To simplify the design while remain the dispersion properties (therefore the wave trajectory) of the cloak, we multiply ε_r and ε_θ by the value of μ_z and obtain the following reduced set of cloak parameters:

$$\mu_z = 1,\quad \varepsilon_\theta = \left(\frac{b}{b-a}\right)^2,\quad \varepsilon_r = \left(\frac{b}{b-a}\right)^2 \left(\frac{r-a}{r}\right)^2. \qquad (2)$$

Compared to the cloak with ideal properties as shown in (1), the reduced parameters in Eq. (2) provide the same wave trajectory. The only adverse effect of using the reduced set is that the impedance at the outer boundary is not perfectly matched and hence some scattering will exist [3]. The level of power reflection due to the reduced parameters can be estimated as $[R_{ab}/(2-R_{ab})]^2$ with $R_{ab} = a/b$ being the ratio between the inner and outer radii.

The non-magnetic nature of the system as indicated in Eq. (2) removes the most challenging issue of the design. The azimuthal permittivity ε_θ is a constant larger than 1, which can be easily achieved in conventional dielectrics. The key to the implementation is to construct the cylindrical shell with the desired radial distribution of ε_r varying from 0 at the inner boundary of the cloak ($r=a$) to 1 at the outer surface ($r=b$). In our design, the required distribution of ε_r is realized by using metal wires of subwavelength size in the radial direction embedded in a dielectric material, as shown in Fig. 1b.The aspect ratio of the metal wires, defined by the ratio of the length to the radius of the wire, is denoted by α. The electromagnetic response of such metal wires can be characterized by the screening factor $\kappa(\alpha)$, which indicates the strength of the interaction between the field and the wire. The effective permittivity ε_{eff} for a composite material comprising metal particles with permittivity ε_m, a volume filling factor f and screening factor

$\kappa(\alpha)$, along with a dielectric component with permittivity ε_d and a filling factor 1-f, is given by the shape-dependent effective medium theory (EMT) [4] as follows:

$$\varepsilon_{eff} = \frac{1}{2\kappa}\left\{\bar{\varepsilon} \pm \sqrt{\bar{\varepsilon}^2 + 4\kappa\varepsilon_m\varepsilon_d}\right\} \qquad (3)$$

where $\bar{\varepsilon} = [(\kappa+1)f - 1]\varepsilon_m + [\kappa - (\kappa+1)f]\varepsilon_d$.

The benefit of using metal wires in a composite cloak is that the radial permittivity ε_r determined by (3) may exhibit a positive value less than 1 with minimal imaginary part. For the structure in Fig. 1b, the filling fraction in (3) for calculating ε_r is $f(r)=f_a\cdot(a/r)$, with f_a being filling fraction at the inner surface of the cloak. The azimuthal permittivity ε_θ inside the cloak is essentially the same as that of the dielectric because the response of the wires to the angular electrical field E_θ oriented normal to the wires is small and at low metal filling factors it can be neglected. The reduced set of cloak parameters in (2) requires a smooth variation of the radial permittivity from 0 to 1 as r changes from a to b. Here we define two filling fraction functions $f_0(\lambda,\alpha)$ and $f_1(\lambda,\alpha)$ such that for given constituent composite materials and a wire aspect ratio of α, the effective permittivity in the radial direction $\varepsilon_{eff,r}(\lambda, f_0(\lambda,\alpha))=0$ and $\varepsilon_{eff,r}(\lambda, f_1(\lambda,\alpha))=1$. Therefore we have $R_{ab}=a/b= f_1(\lambda,\alpha)/f_0(\lambda,\alpha)$. Using this relation together with the expression for ε_θ in (2), we obtain the operating condition of the cloak:

$$\varepsilon_\theta(\lambda) = \left(\frac{f_0(\lambda,\alpha)}{f_0(\lambda,\alpha) - f_1(\lambda,\alpha)}\right)^2 \qquad (4)$$

Where $\varepsilon_\theta(\lambda)$ is the permittivity of the dielectric material surrounding the metal wires in the cloak.

For practical applications, it is important to design a cloaking device operating at a pre-set operational wavelength λ_{op}. For this purpose the design process is as follows. First we choose materials for the metal wires and the surrounding dielectric. Second, we calculate the values of f_0 and f_1 as functions of the aspect ratio α at λ_{op} using the EMT model. The required aspect ratio for λ_{op} is the one that satisfies Eq. (4). Then, the geometrical factors of the cloak, including R_{ab} and f_a, can be determined accordingly. Note that the same design works for all similar cylindrical cloaks with the same shape factor R_{ab}.

3. Implementations and simulations

As a practical example, we have designed an optical cloak operating at λ_{op}=632.8 nm (He-Ne laser) and consisting of silver and silica. The design procedure yields the desired aspect ratio α = 10.7, the shape factor of the cylindrical cloak R_{ab}=0.314, and the volume filling fractions at the two boundaries f_a = 0.125 and f_b = 0.039, respectively. The effective parameters of μ_z, ε_r and ε_θ from this design together with the exact set of reduced parameters determined by Eq. 2 are shown in Fig. 2. We can see that μ_z and ε_θ perfectly match the theoretical requirements throughout the cylindrical cloak. The radial permittivity ε_r fits the values required by Eq. 2 exactly at the two boundaries of the cloak, and follows the overall tendency very well inside the cloak.

To validate if the required distribution of permittivity could be achieved using prolate spheroidal silver nanowires embedded in a silica tube, we determine the effective anisotropic permittivity of a unit cell with sub-wavelength dimensions. The 3D full-wave simulations with finite-element (FE) solver COMSOL MULTIPHYSICS confirm that the range of distributions of ε_{eff} fits rather well with those predicted by EMT, with a discrepancy of around 10%. As for the loss feature, the FE simulations show that the radial permittivity ε_r has an imaginary part of about 0.1 throughout the cloak, which is a very small value for metal-dielectric metamaterials.

To illustrate the performance of the proposed non-magnetic optical cloak with a design corresponding to Fig. 2 and operating at λ_{op} = 632.8 nm, we performed finite-element field mapping simulations for magnetic field distribution around the cloaked object. The object hidden inside the cloak is an ideal metallic cylinder with radius r = a. We note that the size of the cloak is more than six times the operational wavelength, while the simulated area is more than 20 times the wavelength. Fig. 3a shows the field distribution around the metal cylinder surrounded by the designed cloak with parameters given by the diamond markers in Fig. 2. With the cloak (Fig. 3a) the wave fronts flow around the cloaked region with remarkably small perturbation, while without the cloak (Fig. 3b) the waves around the object are severely distorted and an evident shadow is cast behind the cylinder. These simulations clearly show the capability of reducing the scattering from the object hidden inside the cloaked region.

4. Conclusions

We have demonstrated a design of an optical cloak based on coordinate transformation. The non-magnetic nature of our design eases the pain of constructing gradient magnetic metamaterials in 3D space, and therefore paves the way for the realization of cloaking devices at optical frequencies. The proposed design can be generalized to cloaks with other metal structures, such as chains of metal nanoparticles or arrays of shaped metal wires. It can be also adopted for other than the optical spectral ranges, including the infrared and the microwave.

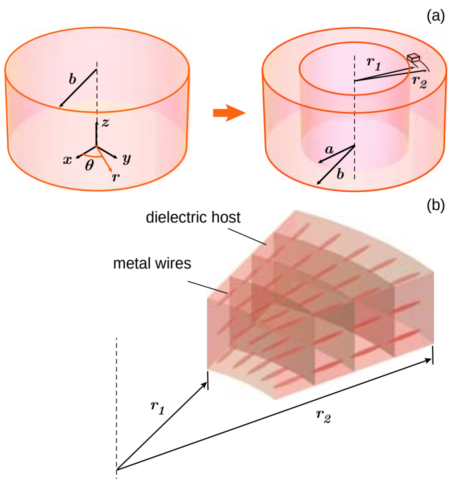

Fig. 1. (a) The coordinate transformation that compresses a cylindrical region $r < b$ into a concentric cylindrical shell $a < r < b$. (b) A small fraction of the cylindrical cloak.

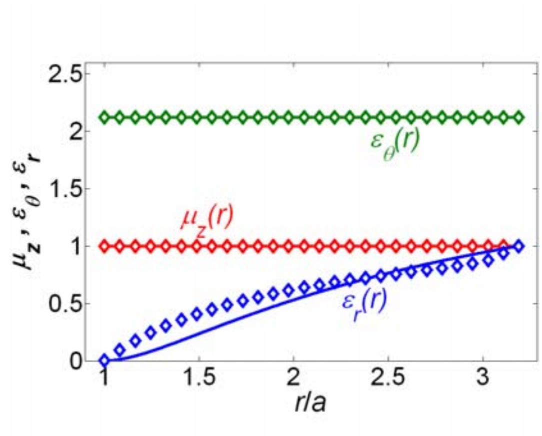

Fig .2 Material parameters ε_r, ε_θ and μ_z of the proposed cloak operating at λ = 632.8 nm. —: parameters determined by eq. (2). ◊: properties of the designed metal wire composite cloak.

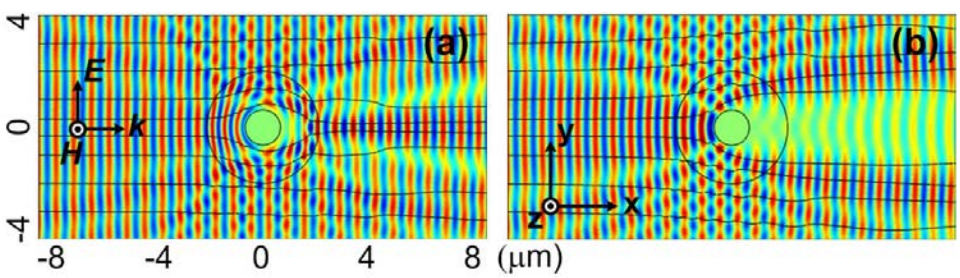

Fig. 3: Full-wave finite-element simulations of the magnetic field mapping around an ideal metallic cylinder of $r = a$ with TM illumination at λ = 632.8 nm. (a) Cloak ON; (b) Cloak OFF.

References

[1] J. B. Pendry, D. Schurig and D. R. Smith, "Controlling electromagnetic fields", *Science* **312**, 1780-1782 (2006).
[2] U. Leonhardt, "Optical conformal mapping", *Science* **312**, 1777-1780 (2006).
[3] D. Schurig, *et al.* "Metamaterial electromagnetic cloak at microwave frequencies", *Science* **314**, 977-980 (2006).
[4] D. E. Aspnes, "Optical properties of thin films", *Thin Solid Films* **89**, 249-262 (1982).